

Influence of the Kondo effect in a non-Fermi liquid system

Karyn Le Hur

Laboratoire de Physique des Solides, Université Paris-Sud, Bât. 510, 91405 Orsay, France

The Kondo effect in a Tomonaga-Luttinger liquid ($U \ll t$) is studied by using the non-Abelian bosonization. The $q = 2k_F$ enhanced spin fluctuations generate a special Kondo effect, for any sign of the exchange coupling J_K ($|J_K| \ll U$) with the impurity. Then, unlike in Fermi liquids ($U \rightarrow 0$), the presence of a $2k_F$ -polarized screening cloud around the impurity favors the occurrence of irrelevant electronic operators with scaling dimension $d = 3/2$: the thermodynamics is nearly this of the two-channel Kondo model in a Fermi liquid. The Mott insulating transition does not affect much the ground state, but rescales the power-law dependence of the Kondo temperature on J_K .

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The antiferromagnetic exchange interaction J_K between a *free* electron gas and a localized magnetic impurity gives rise to the so-called Kondo effect [1]. When the temperature reaches the characteristic Kondo energy scale $T_k \propto e^{-1/\rho J_K}$, where ρ is the density of states per spin, the local moment forms a singlet pair with a conduction electron within an energy T_k of the Fermi energy [2]. In the scaling language, the electrons participating in the formation of this local singlet lie within a characteristic distance $\xi_{imp} \sim v_F/T_k$ where v_F is the Fermi energy. This is a length that is far greater than the lattice spacing and it tends to form a real “screening cloud” around the impurity site. The low-temperature behavior seems trivial. The impurity has disappeared from the low-energy physics, but certain interactions between electrons are generated in the processes of eliminating the impurity spin. The Kondo effect can be mapped to a one-dimensional model, and much of the interesting behavior comes from **two** leading irrelevant electronic operators, which have exactly the same scaling dimension $d = 2$. Nozières [3] argued that they have an universal ratio, so there is only one unknown parameter, the “Wilson number”. We expect the concerned irrelevant coupling constant to be $\mathcal{O}(1/T_k)$ by a standard scaling argument. The first order perturbation theory in it, gives a *heavy-fermion* behavior defined by a linear specific heat $C_{imp} \sim T/T_k$, a constant susceptibility $\chi_{imp} \sim T/T_k$ and an universal Wilson ratio $R_W = 2$.

In three dimensions, electron-electron interactions do not affect the Kondo fixed point, just dress up bare electrons into nearly independent quasiparticles. On the other hand, the Landau’s Fermi liquid theory breaks down in a one dimensional electron gas, for *arbitrarily* weak interactions U [4]. The effect of electron-electron interactions is particularly strong in one dimension and leads to a non-Fermi liquid state (the so-called Tomonaga-Luttinger (TL) liquid [5]). In the presence of repulsive interactions, quasiparticle like excitations are replaced by collective density spin and charge excitations, namely *spinons* and *holons*. Recently, considerable attention has been focused theoretically on the response

of such a system to localized perturbations [6]; it would be experimentally accessible with narrow single-channel quantum wires. Classically, the TL liquid is governed by $q = 2k_F$ charge and especially spin density waves; so, we are led to think that the Kondo effect in a TL liquid is totally different from that in a Fermi liquid. Now, it has been well established that sufficiently enhanced $q = 2k_F$ spin fluctuations also support a special Kondo effect, for ferromagnetic as well as antiferromagnetic Kondo exchanges [7,8]. The Kondo temperature is expected to yield a power-law dependence on the exchange coupling ($|J_K| < U$), due to the particular density of states of the TL liquid in the vicinity of the fixed point. But, the different used weak-coupling analyses [7,8] do not allow to give a precise sight of the ground state in the regime $U > J_K$. Recent exact results from Conformal Field Theory (CFT) [9] are available only in the limit $U < J_K$. In this Letter, we address the following questions: What is the fixed point of the Kondo model in such a non-Fermi liquid system for $U > J_K$? What are the pseudo-particles in the “screening cloud”?

Our starting point is the Hamiltonian:

$$H = -t \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i+1,\sigma} + (h.c) + U \sum_{i,\sigma} n_{i,\sigma}^c n_{i,-\sigma}^c \quad (1)$$

$$+ J_K c_{i=0,\alpha}^\dagger (\vec{\sigma}_\beta^\alpha / 2) c_{i=0,\beta} \vec{\tau}$$

where the bare parameters obey $(U, J_K) \ll t$. Here, $c_{i,\sigma}^\dagger$ ($c_{i,\sigma}$) creates (annihilates) an electron of spin σ at site i and $\vec{\tau}$ is a spin $\frac{1}{2}$ operator located at site $i = 0$. A small U-Hubbard interaction between c-electrons has been introduced, and J_K describes the Kondo coupling. To preserve the SU(2) spin symmetry, to include the effects of the $4k_F$ -Umklapps at half-filling, and to analyze precisely all the irrelevant operators, we use a continuum limit of the Hamiltonian and we switch over to non-Abelian bosonization notations [10].

Non-Abelian notations of the (TL) liquid: We linearize the dispersion of conduction electrons; the lattice step is fixed to $a = 1$. The relativistic fermions $c_\sigma(x)$ are separated in left-movers $c_{L\sigma}(x)$ and right-movers $c_{R\sigma}(x)$ on

the Fermi-cone. As usual, we introduce the “normal-ordered” current operators for the charge and spin degrees of freedom, namely $J_{c,L} =: c_{L\sigma}^\dagger c_{L\sigma}(x) :$ and $\vec{J}_{c,L} =: c_{L\alpha}^\dagger \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{L\beta} :$ and similarly for the right-movers. The charge Hamiltonian is equivalently described in terms of the massless scalar field Φ_c and its moment conjugate Π_c :

$$H_c = \int dx \frac{u_\rho}{2K_\rho} : (\partial_x \Phi_c)^2 : + \frac{u_\rho K_\rho}{2} : (\Pi_c)^2 : \quad (2)$$

$$+ g_3 \exp(i4k_F x) \cos(\sqrt{8\pi}\Phi_c)$$

The coupling $g_3 \propto U$ generates the usual $4k_F$ -Umklapp process *only* relevant at half-filling, and the parameters u_ρ and K_ρ which describe the TL liquid are given by:

$$u_\rho K_\rho = v_F \quad \text{and} \quad \frac{u_\rho}{K_\rho} = v_F + 2U/\pi \quad (3)$$

The Fermi velocity is $v_F = 2t \sin k_F$. The spin sector is described by a $k=1$ Wess-Zumino-Witten (WZW) Hamiltonian [11]:

$$H_s = \frac{2\pi v_F}{3} \int dx : \vec{J}_{c,L}(x) \vec{J}_{c,L}(x) : + (L \rightarrow R) \quad (4)$$

To treat the local Kondo interaction, we need the complete representation for the conduction spin operator [10]:

$$\vec{S}_c \simeq \vec{J}_{c,L}(x) + \vec{J}_{c,R}(x) \quad (5)$$

$$+ \alpha \exp(i2k_F x) \text{tr}(g \cdot \vec{\sigma})(x) \cos(\sqrt{2\pi}\Phi_c)(x)$$

where α is a simple constant. For the following, we do not need explicitly the exact representation of the current operator $\vec{J}_{c,L}(x)$ in terms of the spin matrix g . Finally, the two relevant spin couplings come out as:

$$\lambda_2 (\vec{J}_{c,L}(0) + \vec{J}_{c,R}(0)) \vec{\tau} \quad (6)$$

$$+ \lambda_3 \text{tr}(g \cdot \vec{\sigma})(0) \cdot \cos(\sqrt{2\pi}\Phi_c)(0) \vec{\tau}$$

where $\lambda_{i=2,3} \propto J_K$. The couplings λ_2 and λ_3 respectively generate the *forward* and *backward* Kondo scattering processes. One aim of this paper is to clarify the fact, that a heavy-fermion fixed point cannot occur in a TL liquid, so we start with the bare constraint $J_K \ll U$. The effects of the electronic interactions are crucial in that case.

Luttinger liquid away from half-filling: We first re-investigate the weak-coupling regime in which $J_K/\pi v_F \ll U/\pi v_F \ll 1$, preserving explicitly the $SU(2)$ symmetry. By using the following commutation rules ($z = x + v_F t$):

$$[\mathcal{J}_{c,L}^a(z), \mathcal{J}_{c,L}^b(z')] = i f^{abc} \mathcal{J}_{c,L}^c(z) \delta(z - z') \quad (7)$$

$$+ \frac{i}{4\pi} \delta^{ab} \delta'(z - z')$$

with $\mathcal{J}_{c,L}^a = J_{c,L}^a \delta(x)$, and the integral: $\frac{1}{v_F} \int \frac{du}{u} \sim \frac{\ln L}{v_F}$ with $u = v_F t$, we obtain the usual λ_2 -beta function [10]:

$$\beta(\lambda_2) = \frac{d\lambda_2}{d\ln L} = \frac{\lambda_2^2}{2\pi v_F} \quad (8)$$

It learns, that λ_2 scales to strong couplings, at the standard Kondo energy scale: $T_k^{(2)} \simeq E_o e^{-2\pi v_F/J_K}$, where E_o is the bare bandwidth cut-off. To obtain the precise recursion relation of the term λ_3 , we consider a given static configuration of the impurity, characterized by the c-numbers $\{\tau^a\} = \pm 1/2$ ($a=x,y,z$). Then, the partition function's invariance under the cut-off transformation $a = 1 \rightarrow a' = e^{d\ln L}$ simply imposes that $\lambda_3^2(a') = \lambda_3^2(a=1)(\frac{1}{a'})^{(1-K_\rho)}$; it gives the more relevant equation:

$$\beta(\lambda_3) = \frac{d\lambda_3}{d\ln L} = \frac{1}{2}(1 - K_\rho)\lambda_3 = \frac{U\lambda_3}{2\pi v_F} \quad (9)$$

Finally, λ_3 is well expected to become strong, at the temperature scale [7,8]:

$$T_k^{(3)} \simeq E_o \cdot \left(\frac{J_K}{U}\right)^{2\pi v_F/U} \gg T_k^{(2)} \quad (10)$$

We distinguish well two different regimes. When $U \rightarrow 0$, the physics is ruled by the forward Kondo scattering term, giving rise to a heavy-fermion behavior [10]. Conversely, when $U \gg J_K$, we may check that the Kondo temperature has a power-law dependence on the coupling exchange J_K , but also that the quenching of the impurity moment occurs for *ferromagnetic* as well as *antiferromagnetic* backward Kondo scattering λ_3 . $T_k^{(3)}$ does not change under $\lambda_3 \rightarrow -\lambda_3$. The most remarkable feature in the case of a ferromagnetic exchange J_K is that a bound singlet state is formed at the impurity site, whatever the small given Ising anisotropy of type: $|\Delta_z| \propto (\lambda_{2\perp} - \lambda_{2z}) \ll 1$; indeed, the energy scale to create a spin $S=1$ in the Luttinger liquid would be:

$$T_{S=1} \simeq E_o \cdot e^{-\text{constant}/\sqrt{|\Delta_z|}} \ll T_k^{(3)} \quad (11)$$

It is finally due to the special nature of the spinons with $S=1/2$, which *cannot* accept the emergence of a spin $S=1$ in the TL liquid. It supports, that the presence of the Kondo backward scattering term accounts for the fundamental difference between a Kondo effect in a TL liquid and that in a Fermi liquid. Unlike the ref. [8], the present formalism shows explicitly, that the recursion relations of λ_2 and λ_3 are really independent up to the orders J_K^2 and UJ_K . In the weak-coupling regime ($T \gg T_k^{(3)}$), the thermodynamic properties of the TL liquid, which vary as L/v_F (where L is the size of the sample) are not affected by the presence of the located impurity $\vec{\tau}$. However, since the term λ_3 changes the direction of propagation of the particles, it certainly modifies the universal conductance of the TL liquid $G_o = 2e^2 K_\rho/h$, obtained by applying a static field over a finite part of the sample. Whereas a key quantity for the Kondo effect in 3D is

the resistivity, it is the conductance in 1D. In the weak-coupling regime, the small coefficient of reflection imposed by the impurity is defined as $\mathcal{R} \sim \lambda_3^2 = G_o - G(T)$. Using Eq.(9), it suggests the following thermal correction: $G_o - G(T) \sim T^{K_\rho - 1}$.

Strong coupling limit: As in the usual Kondo effect ($U \rightarrow 0$), we have to consider the effect of all *irrelevant* operators. They might be different because here the $q = 0$ spin density is not involved in the strong coupling description. We work in the infinite length limit and re-consider the conductance, specific heat and susceptibility. Naively, the formation of a spin singlet at the impurity site separates the TL liquid into two semi-infinite pieces. Unlike in Fermi liquids, it can produce exotic tunnelling phenomena in TL ones [6,8](cf Fig 2a). The **first** irrelevant operator would come from the process in which an electron tunnels from one TL liquid to the other, with virtually breaking the spin singlet. The appropriate tunnelling is of the form

$$\begin{aligned} \delta H_{\text{barrier}} &= t_o \sum_{\alpha} [c_{1\alpha}^\dagger(x=0)c_{2\alpha}(x=0) + (h.c)] \quad (12) \\ &= t_o \cos[\sqrt{2\pi}\tilde{\Phi}_c(0)] \cdot \text{tr}g(0) \end{aligned}$$

where $c_1(c_2)$ is the electron operator in the left (right) semi-infinite TL liquid. Here, $t_o \sim 1/J_K^*$ denotes the bare tunnelling amplitude.

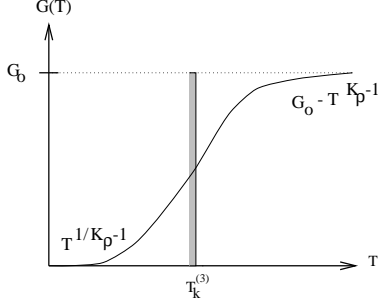


FIG. 1. Behavior of the conductance in the presence of the impurity moment.

The operators have to satisfy the *fixed boundary condition* $c_i(x=0) = 0$, different from the periodic ones we have used before. This can be achieved by fixing $\Phi_c(x=0) = \sqrt{\pi/2}$ [12], and by using the irrelevant *dual* charge field $\tilde{\Phi}_c = \frac{1}{2}[\tilde{\Phi}_{c1} - \tilde{\Phi}_{c2}]$ and the spin matrix $g = \frac{1}{2}[g_1 - g_2]$. Then, t_o obeys a flow equation of type of Eq.(9), but now replacing $K_\rho \rightarrow 1/K_\rho$. At zero temperature, it vanishes; this is a reflection of the suppressed density of states in a Luttinger liquid. Since the tunnelling perturbation t_o is *irrelevant*, the conductance varies as $G(T) \sim t_o^2(T) \sim t_o^2 T^{(1/K_\rho)-1}$. More generally, we can consider the contribution to the impurity free energy of $\mathcal{O}(t_o^2)$. A simple rescaling argument implies that $f_{\text{imp}} \propto T^{1+2\Delta}$, where $(\Delta + 1)$ is the dimension of the irrelevant operator. Explicitly, $\Delta = 1/2(1/K_\rho - 1)$, and we may

confirm [8], that it provokes deviations from the linear behavior of the specific heat, $C_{\text{imp}}^{(1)} \propto T^{2\Delta} \sim T^{(1/K_\rho)-1}$. The susceptibility is not corrected by processes in $\mathcal{O}(t_o^2)$, since the impurity remains completely screened.

But now, unlike ref. [8], we argue that the Kondo effect in a TL liquid should generate other more crucial irrelevant operators than $t_o^* \rightarrow 0$. The impurity may be screened by any electron at a distance $x \leq \xi_{\text{imp}} \sim v_F/T_k^{(3)}$ of it (cf Fig 2b), and ξ_{imp} is considerably larger than $a = 1$. Since we study the physics at a length $L = 1/|k - k_F| \gg \xi_{\text{imp}}$, the physics in the screening cloud can be still modeled by $\delta(x)$ -interactions. Then, since $t_o^* \rightarrow 0$ or equivalently $\langle \cos[\sqrt{2\pi}\Phi_c(0)] \rangle \neq 0$, the $q = 2k_F$ fluctuations diverge around the impurity site when $T \rightarrow 0$. ξ_{imp} has not the same meaning as in a Fermi liquid: here, it occurs as the *pinning-length* of the $q = 2k_F$ spin density wave $O_{SDW} = [c_{L\uparrow}^\dagger c_{R\downarrow}]$ by the impurity moment.

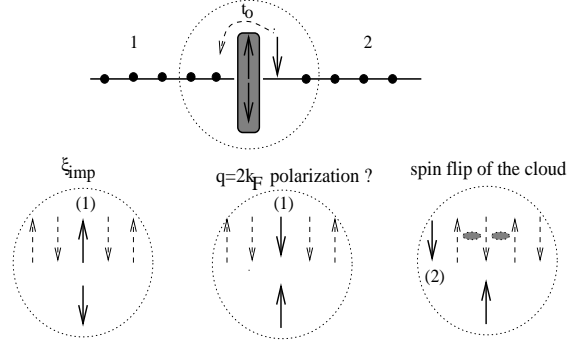


FIG. 2. (a) The formation of a spin singlet might lead to a possible tunnelling process; but t_o vanishes at the fixed point. (b) Then, the $q = 2k_F$ spin fluctuations coupled to the impurity diverge in the screening cloud, that engenders strong correlations between an electron (1) which flips the impurity spin and its two neighbors.

In fact, the low-energy description in terms of spinons, should definitely fail in the screening cloud due to prominent spin flips with the impurity. In the original Kondo effect, the impurity spin changes the bare free electrons into free heavy-fermions. In TL liquids, we argue that it would change weakly repulsive electrons into heavy pseudo-particles, giving another generic non-Fermi liquid behavior.

What are the other irrelevant operators allowed by symmetry? Unlike in Fermi liquids [10], we can remark that no electron-electron interaction in $\eta = 1/T_k^{(3)}$ may be induced by two respective *spin flips* with the impurity. Indeed, it gives the charge term $\eta_1 c_{L\uparrow}^\dagger c_{R\downarrow} c_{R\downarrow}^\dagger c_{L\uparrow}(x=0) = -\eta_1 J_{c,L} J_{c,R} \delta(x)$, and the spin current term $-\eta_2 c_{L\uparrow}^\dagger c_{R\uparrow} c_{R\downarrow}^\dagger c_{L\downarrow}(x=0) = \eta_2 \vec{J}_{c,L} \vec{J}_{c,R} \delta(x)$. But, $\eta_1 \sim 1/U$, since U defines the $q = 0$ -interaction energy scale in the charge sector and $\eta_2 = 0$ since here the $q = 0$ spin

density is not involved in the Kondo effect.

Typical two-channel Kondo operator: But, if we still parallel the well-known irrelevant operators of the standard Kondo effect in a Fermi liquid, we have to add the following term

$$\begin{aligned}\delta H_{2channel} &= \eta_3 \frac{d}{dx} [c_{L\alpha}^\dagger(x=0) c_{R\alpha}(x=0)] \\ &= \eta_3 \frac{d}{dx} \text{tr} g(x=0) \quad \eta_3 \sim 1/\sqrt{T_k^{(3)}}\end{aligned}\quad (13)$$

which respects site Parity and which is obtained through a $1/T_k^{(3)}$ -expansion. In fact, such an operator which has the scaling dimension $3/2$ is known to be very important in a periodic Heisenberg chain by varying two adjacent sites by the same amount [12]. In the present case, when an electron (1) comes from flipping the impurity spin, it makes all the spin environment unstable because disturbs the $q = 2k_F$ magnetic order (cf Fig 2b). *Simultaneously*, the other electrons in the screening cloud should also flip their spins in view to restore the $q = 2k_F$ antiferromagnetic polarization, strongly coupled to the impurity spin: these new collective spin excitations are not spinons. This should generate the expected *strong* antiferromagnetic links defined by $\eta_3 \sim 1/\sqrt{T_k^{(3)}}$ between the electron (1) and its two neighbors. In fact, this leading irrelevant operator can also be written as $(\vec{J}_{L,-1} + \vec{J}_{R,-1}) \cdot \text{tr} g \vec{\sigma}$ [12] ($\vec{J}_{L,-1}$ is the first Kac-Moody descendent [10]), and rewriting the theory only with left movers, this becomes equivalent to a single $k=2$ WZW field \vec{J}_{-1} with a spin-1 primary field $\vec{\phi}$: $\eta_3 \vec{J}_{-1} \cdot \vec{\phi}$ [9]. We realize that η_3 is the same irrelevant operator as in the usual two-channel Kondo model [10,13], and from second-order perturbation theory, it leads:

$$C_{imp}^{(2)} \propto \frac{T}{T_k^{(3)}} \ln \frac{T}{T_k^{(3)}}, \quad \chi_{imp}^{(2)} \propto \ln \frac{T}{T_k^{(3)}} \quad (14)$$

However, the Wilson ration R_W cannot be universal because here, there are two irrelevant operators in the theory. The low-energy behavior is nearly the same as for the two-channel Kondo model in a free electron gas.

Mott U-transition: Umklapps are responsible for the Mott U-transition at half-filling [14]. If we define incommensurability q such as $2(k_F + q) = \pi$, the charge sector becomes massive for $q \rightarrow q_c \sim \pi/2L_c$ with the soliton length $L_c \sim \frac{1}{\sqrt{8\pi}} \sqrt{\pi v_F/U}$. It gives:

$$\frac{dg_3}{d\ln L} = 2(1 - K_\rho)g_3 \quad (15)$$

The resulting charge gap is of the form $\Delta_c \propto E_o e^{-\pi v_F/U}$. Now, there is an energy gap $T \sim \Delta_c$ for creation of kinks, which carry $S=0$ and $Q=2e$. Then, for $T \ll \Delta_c$, we must carefully replace $\langle \cos[\sqrt{2\pi}\Phi_c(x)] \rangle \sim \text{constant}$ and the renormalized exponent K_ρ^* tends to $1/2$. Since

Umklapps enhance considerably the $q = 2k_F$ spin density wave, we obtain:

$$\frac{d\lambda_3}{d\ln L} = \frac{1}{2}(1 - \frac{1}{2})\lambda_3 \quad (16)$$

In that context, the Kondo temperature is rescaled to $T_k^{(3)} \propto E_o(J_K/U)^4 \ll \Delta_c$. It may be related to the problem of a spin defect in a spin-1/2 Heisenberg chain. But, $\xi_{imp} \sim a = 1$ due to the particular value of the bare interaction $U \rightarrow +\infty$ and then, the screening cloud has no meaning [12]. Here, ξ_{imp} is still much larger than a : quantum fluctuations are reduced but subsist. The *single* permitted irrelevant operator is η_3 , and we obtain a Wilson ratio $R_W = 4/3(1 + u_\rho^*/v_F) \rightarrow 4/3$.

Summarizing, a magnetic impurity has a great influence on TL liquids ruled by $U > J_K$. Although the impurity spin is screened, irrelevant electronic interactions may change the collective density spin excitations of the TL liquid into heavy “pseudo-particles”, giving the same generic non-Fermi liquid behavior as the two-channel Kondo model in a Fermi liquid.

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